Humphrey

Voltages Induced Upon Telephone Lines Near Three Phase Transmission Lines

VOLTAGES INDUCED UPON TELEPHONE LINES NEAR THREE PHASE TRANSMISSION LINES

BY

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Head of Department of Ellery B. Paine

Recommendation concurred in:

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VOLTAGES INDUCED UPON

TELEPHONE LINES

NEAR THREE PHASE TRANSMISSION LINES

MAGNETIC

By the current in each wire of the transmission line there is produced a magnetic flux surrounding that wire. Where spacings are large and wire diameters small, as is always the case in high voltage transmission, this flux consists of circular lines of force, the centers of these circles of force being the center of the wire carrying the current which causes them. That is, the flux surrounding each conductor is the same as it would be if it were carrying the same current and the other conductors were not present. For this case, the flux density is $0.2\frac{i}{x}$ at a point distant x from the center of the wire carrying the current i amperes. And the flux between points distant x_1 and x_2 respectively is

$$\int_{x_2}^{x_1} 0.2\frac{i}{x}dx = 0.2 \ln\frac{x_1}{x_2}$$

It appears that if x_1 be infinite, then the total flux is also infinite.* However, this infinite flux may in all practical cases be divided into an effective finite flux and an infinite flux which is exactly destroyed by a counteracting infinite flux.

Fig. 1 represents in cross section the three wires of a transmission line, A, B, and C, and the accompanying telephone line, T. Let us consider the flux surrounding A. If the current in A is I_A, the flux surrounding it is:

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* This is not, of course, true, but appears due to assumption of an infinite length of wire.

$$\phi_1 = 0.2 I_A \ln \frac{\infty}{r}$$

r being wire radius, ϕ_1 is flux external to the wire. But of this total flux surrounding A_1 a certain part we know does not cut the telephone, as it lies inside the distance a. We may say, then, that the flux which cuts the wire T due to the current in A is

$$\phi_{t_1} = (0.2 \ln\frac{\infty}{r} - 0.2 \ln\frac{a}{r})\ I_A$$

$$= (0.2 \ln\frac{\infty}{a})\ I_A$$

Obviously only this part of the flux can have any effect in producing voltage in T.

The currents in B and C also produce fluxes, and of these the parts which cut T are

$$\phi_{t_2} = 0.2\ I_B \ln\frac{\infty}{b}$$

$$\phi_{t_3} = 0.2\ I_C \ln\frac{\infty}{c}$$

and the total flux cutting T is the sum of these

$$\phi_t = 0.2\left\{ I_A \ln\frac{\infty}{a} + I_B \ln\frac{\infty}{b} + I_C \ln\frac{\infty}{c} \right\} \qquad (1)$$

For the three phase condition we may assume

$$\begin{aligned} I_A &= I \sin\omega t \\ I_B &= I \sin(\omega t + 120^\circ) \\ I_C &= I \sin(\omega t + 240^\circ) \end{aligned} \qquad (2)$$

where I is the maximum value of current in any wire. Were the phases unbalanced, other equations connecting I_A, etc., with time should be used. These would, however, introduce nothing new in the method of treatment, which is sufficiently general if the balanced case alone be considered.

Substituting (2) in (1)

$$\phi_t = 0.2I\left\{ \ln\frac{\infty}{a} \sin\omega t + \ln\frac{\infty}{b}\sin(\omega t + 120^\circ) + \ln\frac{\infty}{c}\sin(\omega t+240^\circ) \right\}$$

$$= 0.2I\left\{ \ln\frac{\infty}{a}\sin\omega t + \ln\frac{\infty}{a}\cdot\frac{a}{b}\sin(\omega t+120^\circ) + \right.$$

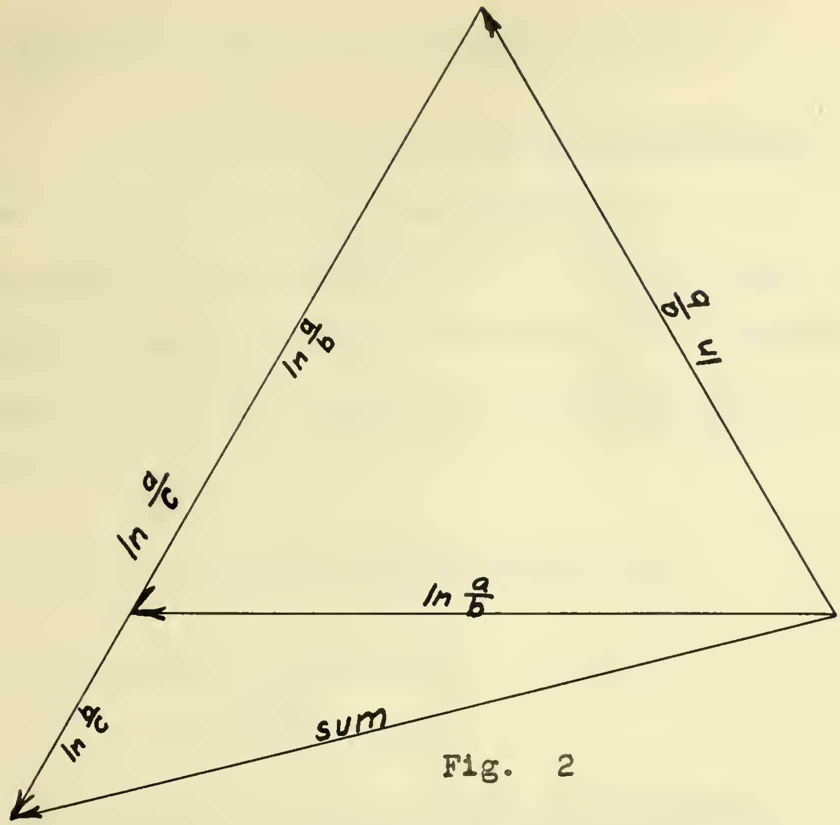

Fig. 2

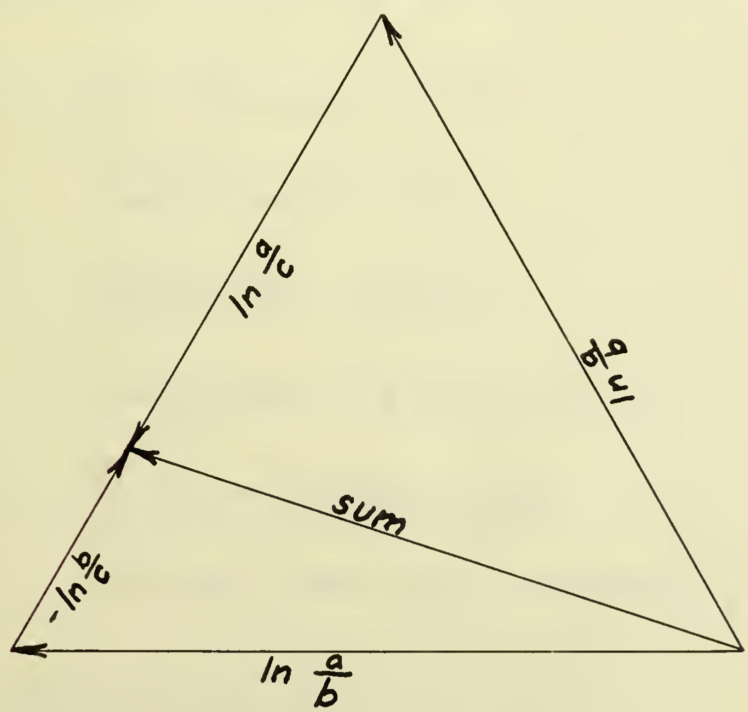

Fig. 3

$$\ln\frac{\infty}{a}\cdot\frac{a}{c}\sin(\omega t+240^\circ)\Big\}$$

$$= 0.2I\left(\ln\frac{\infty}{a}\Big\{\sin\omega t+\sin(\omega t+120^\circ)+\sin(\omega t+240^\circ)\Big\}\right.$$

$$\left.+\ln\frac{a}{b}\sin(\omega t+120^\circ)+\ln\frac{a}{c}\sin(\omega t+240^\circ)\right) \quad (3)$$

A well known theorem tells us that the sum

$I\sin a+I\sin(a+\frac{360^\circ}{n})+I\sin(a+2\frac{360^\circ}{n})$ ······· to n terms is zero irrespective of the value of I and a. With all propriety then this term may be dropped out of (3), leaving as the flux which may produce voltage in T

$$\phi_t = 0.2I\left(\ln\frac{a}{b}\sin(\omega t+120^\circ)+\ln\frac{a}{c}\sin(\omega t+240^\circ)\right) \quad (4)$$

The voltage induced by the flux is

$$e = n\frac{d\phi_1}{dt}\cdot 10^{-8}$$

$$= 0.2I\left(\ln\frac{a}{b}\cos(\omega t+120^\circ)+\ln\frac{a}{c}\cos(\omega t+240^\circ)\right)\cdot 10^{-8}\cdot\omega \quad (5)$$

$$e = 0.2I\left(\ln\frac{a}{b}\cos\omega t\cdot\cos 120^\circ\right.$$

$$-\ln\frac{a}{b}\sin\omega t\cdot\sin 120^\circ$$

$$+\ln\frac{a}{c}\cos\omega t\cdot\cos 240^\circ$$

$$\left.-\ln\frac{a}{c}\sin\omega t\cdot\sin 240^\circ\right)\cdot 10^{-8}\cdot\omega$$

$$= 0.2I\omega\left(\cos\omega t(-\tfrac{1}{2}\ln\frac{a}{b}-\tfrac{1}{2}\ln\frac{a}{c})\right.$$

$$\left.+\sin\omega t(-.866\ln\frac{a}{b}+.866\ln\frac{a}{c})\right)\cdot 10^{-8}$$

$$= 0.2I\omega\left(\sqrt{\tfrac{1}{4}(\ln\frac{a}{b}+\ln\frac{a}{c})^2+\frac{3}{2}(\ln\frac{a}{c}-\ln\frac{a}{b})^2}\right)$$

$$\text{times} \quad \sin\left(\omega t - \arctan\frac{\frac{1}{2}(\ln\frac{a}{b}+\ln\frac{a}{c})}{.866(\ln\frac{a}{c}-\ln\frac{a}{b})}\right)\cdot 10^{-8}$$

$$=0.2I\omega\sqrt{\tfrac{1}{4}(\ln\frac{a}{b}+\ln\frac{a}{c})^2 + \frac{3}{2}(\ln\frac{b}{c})^2}$$

$$\sin\left(\omega t - \arctan\frac{\ln\frac{a}{b}+\ln\frac{a}{c}}{\sqrt{3}\ln\frac{b}{c}}\right)\cdot 10^{-8} \qquad (6)$$

Of this voltage wave the r.m.s value is

$$E_t = \frac{0.2\cdot 2\pi f}{\sqrt{2}} \; I\sqrt{\tfrac{1}{4}(\ln\frac{a}{b}+\ln\frac{a}{c})^2 + \frac{3}{2}(\ln\frac{b}{c})^2}\cdot 10^{-8}$$

Or, if for I we substitute its effective (r.m.s) value, I_e,

$$E_t = 1.257 \; f \; I_e\sqrt{\tfrac{1}{4}(\ln\frac{a}{b}+\ln\frac{a}{c})^2 + \frac{3}{2}(\ln\frac{b}{c})^2}\cdot 10^{-8}\cdot\text{length of line in cm.}$$

This is the voltage between the ends of a single telephone wire. Considering the telephone circuit, this voltage is opposed by a nearly equal one in the other telephone wire, and the difference, though slight, would cause an enormous roar of the transmission frequency. In practice this is avoided by transposition of the telephone wires, making these opposing voltages exactly equal.

For numerical work it will be found easier to add $\ln\frac{a}{b}$ and $\ln\frac{a}{c}$ (see equation 5) graphically at 120°, or, what is the same thing, $\ln\frac{a}{b}$ and $\ln\frac{b}{c}$ at 60°.

Probably $\ln\frac{b}{c}$ will generally be negative (it is so in Fig. 1), when it must be measured up and to the right on the sloping line: (Fig. 3).

Having obtained this vector sum, equation 7 becomes

$$E_t = 1.257\, I_e f\begin{pmatrix}\text{sum of}\\ \ln\frac{a}{b}\text{ and }\ln\frac{b}{c}\\ \text{at } 10^{\circ}\end{pmatrix}\cdot 10^{-8}\cdot\begin{pmatrix}\text{length of}\\ \text{line in}\\ \text{centimeters}\end{pmatrix} \qquad (7a)$$

Using common logarithms (to base 10)

$$E_t = 2.89\ f\ I_e \sqrt{\tfrac{1}{4}(\log\frac{a}{b}+\log\frac{a}{c}) + \frac{3}{2}(\log\frac{b}{c})^2} \cdot 10^{-3} \cdot \begin{pmatrix}\text{length}\\ \text{line}\\ \text{in km.}\end{pmatrix} \qquad (7)$$

$$= 4.63\ f\ I_e \begin{pmatrix}\text{sum of}\\ \log\frac{a}{b}\ \&\ \log\frac{b}{c}\\ \end{pmatrix} \cdot 10^{-3} \cdot \begin{pmatrix}\text{length}\\ \text{line in}\\ \text{miles}\end{pmatrix} \qquad (7a)$$

DIELECTRIC

In considering the voltage produced by electrostatic induction there are many cases to be considered, depending upon the connection or non-connection of the system to the earth and upon the presence of ground wires. In the magnetic case these made no appreciable difference, since in any condition of ground by far the greater part of the current will flow in the wires, and little in the ground -- so little that the images of the line need not be considered. On the other hand, the potentials of the wires are entirely dependent on the condition of grounding, and with them, the voltages they induce upon the telephone line.

CASE I -- NEUTRAL OF SYSTEM GROUNDED.

This is of course the simplest case. For it is needed a sketch showing in cross section the three transmission wires, with their images, (Fig. 4).

If, as is always the case in high voltage transmission, the spacing and height of wires above ground is great, as compared with their diameters, the center of each conductor and of each image may be considered the axis of a uniform radiating field of dielectric flux, and all effects of electrostatic induction may be thot of as being produced by these radial fields.

In any one of these fields the flux density varies inversely

as the distance from its axis. If the flux is ψ lines of force per centimeter length of axis, the flux density at a distance x is $\frac{\psi}{2\pi x}$. This is proportional to the voltage gradient at that point.

$$\frac{de}{dx} = K_1 \frac{\psi}{2\pi x} \tag{8}$$

The voltage V between points distant x_1 and x_2 is

$$\int_{x_2}^{x_1} de = \frac{K_1\psi}{2\pi}\int_{x_2}^{x_1} \frac{dx}{x}$$

$$= \frac{K_1}{2\pi}\psi \ln\frac{x_1}{x_2}$$

$$= \frac{2.3K_1}{2\pi}\psi \log\frac{x_1}{x_2}$$

$$= K\psi \log\frac{x_1}{x_2} \tag{9}$$

The value of K (or K_1) is easily found, but is not at present necessary.

In Fig. 4 let there be at A, B, C the fluxes ψ_1, ψ_2, ψ_3. At the images A', B', C' there will then be the fluxes $-\psi_1$, $-\psi_2$, $-\psi_3$. Due to these fluxes there are differences of potential between each wire and its image. These are tabulated in Table I.

(Note: Subscript 1 refers to line A, 2 to B, 3 to C, i.e., r_1 is radius of wire A, h_3 is height of C above earth).

Since the neutral is grounded the voltages above ground are

$$\begin{aligned} P_A &= e_m \sin\omega t \\ P_B &= e_m \sin(\omega t + 120^\circ) \\ P_C &= e_m \sin(\omega t + 240^\circ) \end{aligned} \tag{10}$$

The voltage between a wire and its image is of course twice the potential of the wire. With the help of (10) there may be written from Table I

TABLE I

DUE TO		POTENTIAL DIFFERENCE BETWEEN		
flux	at	A & A'	B & B'	C & C'
ψ_1	A	$K\psi_1 \cdot \log\frac{2h_1}{r_1}$	$\log\frac{AB'}{AB}$	$\log\frac{AC'}{AC}$
$-\psi_1$	A'	$-K\psi_1 \cdot \log\frac{r_1}{2h_1}$	$\log\frac{A'B'}{A'B}$	$\log\frac{A'C'}{A'C}$
ψ_2	B	$K\psi_2 \cdot \log\frac{A'B}{AB}$	$\log\frac{2h_2}{r_2}$	$\log\frac{BC'}{BC}$
$-\psi_2$	B'	$-K\psi_2 \cdot \log\frac{A'B'}{AB'}$	$\log\frac{r_2}{2h_2}$	$\log\frac{B'C'}{B'C}$
ψ_3	C	$K\psi_3 \cdot \log\frac{A'C}{AC}$	$\log\frac{B'C}{BC}$	$\log\frac{2h_3}{r_3}$
$-\psi_3$	C'	$-K\psi_3 \cdot \log\frac{A'C'}{AC'}$	$\log\frac{B'C'}{BC'}$	$\log\frac{r_3}{2h_3}$

$$
(11)\qquad
\begin{aligned}
K\psi_1\log\frac{2h_1}{r_1} + K\psi_2\log\frac{A'B}{AB} + K\psi_3\log\frac{A'C}{AC} &= e\sin\omega t\\
K\psi_1\log\frac{AB'}{AB} + K\psi_2\log\frac{2h_2}{r_2} + K\psi_3\log\frac{BC'}{BC} &= e\sin(\omega t + 120^\circ)\\
K\psi_1\log\frac{A'C}{AC} + K\psi_2\log\frac{B'C}{BC} + K\psi_3\log\frac{2h_3}{r_3} &= e\sin(\omega t + 240^\circ)
\end{aligned}
$$

It should be noted that $A'C = AC'$, $A'B' = AB$, etc.

The solution of (11) will give ψ_1, ψ_2, ψ_3. In determinant form this solution is:

Determinant of system

$$
\begin{vmatrix}
\log\frac{2h_1}{r_1} & \log\frac{A'B}{AB} & \log\frac{A'C}{AC}\\
\log\frac{A'B}{AB} & \log\frac{2h_2}{r_2} & \log\frac{B'C}{BC}\\
\log\frac{A'C}{AC} & \log\frac{B'C}{BC} & \log\frac{2h_3}{r_3}
\end{vmatrix}
$$

$$= D_1 \qquad (12)$$

$$K\psi_1 = \frac{e_m \begin{vmatrix} \sin\omega t & \log\frac{A'B}{AB} & \log\frac{A'C}{AC} \\ \sin(\omega t+120^\circ) & \log\frac{2h_2}{r_2} & \log\frac{B'C}{BC} \\ \sin(\omega t+240^\circ) & \log\frac{B'C}{BC} & \log\frac{2h_3}{r_3} \end{vmatrix}}{D_1} \tag{12a}$$

$$K\psi_2 = \frac{e_m \begin{vmatrix} \log\frac{2h_1}{r_1} & \sin\omega t & \log\frac{A'C}{AC} \\ \log\frac{A'B}{AB} & \sin(\omega t+120^\circ) & \log\frac{B'C}{BC} \\ \log\frac{A'C}{AC} & \sin(\omega t+240^\circ) & \log\frac{2h_3}{r_3} \end{vmatrix}}{D_1} \tag{12b}$$

$$K\psi_3 = \frac{e_m \begin{vmatrix} \log\frac{2h_1}{r_1} & \log\frac{A'B}{A\ B} & \sin\omega t \\ \log\frac{A'B}{A\ B} & \log\frac{2h_2}{r_2} & \sin(\omega t+120^\circ) \\ \log\frac{A'C}{A\ C} & \log\frac{B'C}{B\ C} & \sin(\omega t+240^\circ) \end{vmatrix}}{D_1} \tag{12c}$$

Note: e_m is maximum voltage to neutral $= \frac{E\sqrt{2}}{\sqrt{3}}$ if E is r.m.s line voltage

The potential (voltage above ground) of T due to fluxes ψ_1 and $-\psi_1$ at A and A' is

$$K\psi_1 \log\frac{h_1}{a} - K\psi_1 \log\frac{h_1}{a'} = K\psi_1 \log\frac{a'}{a}$$

Having obtained $K\psi_1$, $K\psi_2$, and $K\psi_3$ from 5, the potential of T is then

$$P_T = K\psi_1 \log\frac{a'}{a} + K\psi_2 \log\frac{b'}{b} + K\psi_3 \log\frac{c'}{c} \tag{13}$$

CASE II SYSTEM NON-GROUNDED.

In this case the voltages to ground are unknown, and the voltages between wires must be used. Calling voltage between A and B $E_m \sin\omega t$; that between B and C is $E_m \sin(\omega t+120^\circ)$; and that between C and A is $E_m \sin(\omega t+240^\circ)$. These voltages may be

considered as due to the fluxes at ABC A'B'C' as before. The "partial voltages" are tabulated in Table II

TABLE II

DUE TO		POTENTIAL DIFFERENCE BETWEEN		
flux	at	A - B	B - C	C - A
ψ_1	A	$K\psi_1 \cdot \log\frac{A\,B}{r_1}$	$\log\frac{A\,C}{A\,B}$	$\log\frac{r_1}{A\,C}$
$-\psi_1$	A'	$-K\psi_1 \cdot \log\frac{A'B}{2h_1}$	$\log\frac{A'C}{A'B}$	$\log\frac{2h_1}{A'C}$
ψ_2	B	$K\psi_2 \cdot \log\frac{r_2}{A\,B}$	$\log\frac{B\,C}{r_2}$	$\log\frac{A\,B}{B\,C}$
$-\psi_2$	B'	$-K\psi_2 \cdot \log\frac{2h_2}{AB'}$	$\log\frac{B'C}{2h_2}$	$\log\frac{AB'}{B'C}$
ψ_3	C	$K\psi_3 \cdot \log\frac{B\,C}{A\,C}$	$\log\frac{r_2}{B\,C}$	$\log\frac{A\,C}{r_3}$
$-\psi_3$	C'	$-K\psi_3 \cdot \log\frac{BC'}{AC'}$	$\log\frac{2h_3}{BC'}$	$\log\frac{AC'}{2h_3}$

From table II

$$K\psi_1\log\frac{A\,B}{A'B}\cdot\frac{2h_1}{r_1} + K\psi_2\log\frac{A\,C}{A'C}\cdot\frac{A'B}{A\,B} + K\psi_3\log\frac{r_1}{2h_1}\cdot\frac{A'C}{A\,C} = E_m \sin\omega t$$

$$K\psi_1\log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} + K\psi_2\log\frac{B\,C}{B'C}\cdot\frac{2h_2}{r_2} + K\psi_3\log\frac{A\,B}{AB'}\cdot\frac{B'C}{B\,C} = E_m \sin(\omega t+120^\circ)$$

$$K\psi_1\log\frac{B\,C}{B'C}\cdot\frac{A'C}{A\,C} + K\psi_2\log\frac{r_3}{2h_3}\cdot\frac{B'C}{B\,C} + K\psi_3\log\frac{A\,C}{A'C}\cdot\frac{2h_3}{r_3} = E_m \sin(\omega t+240^\circ) \tag{14}$$

These equations are not independent since the sum of the three line voltages must be zero. Any two may, however, be solved with

$$\psi_1 + \psi_2 + \psi_3 = 0$$

Choosing the first two this solution is:

$$\begin{vmatrix} \log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} & \log\frac{AC}{A'C}\cdot\frac{A'B}{AB} & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} & \log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= D_2 \tag{15}$$

$$K\psi_1 = \frac{E_m \begin{vmatrix} \sin \omega t & \log\frac{AC}{A'C}\cdot\frac{A'B}{AB} & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \sin(\omega t+120^\circ) & \log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \\ 0 & 1 & 1 \end{vmatrix}}{D_2} \tag{15a}$$

$$K\psi_2 = \frac{E_m \begin{vmatrix} \log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} & \sin \omega t & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} & \sin(\omega t+120^\circ) & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \\ 1 & 0 & 1 \end{vmatrix}}{D_2} \tag{15b}$$

$$K\psi_3 = \frac{E_m \begin{vmatrix} \log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} & \log\frac{AC}{A'C}\cdot\frac{A'B}{AB} & \sin \omega t \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} & \log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} & \sin(\omega t+120^\circ) \\ 1 & 1 & 0 \end{vmatrix}}{D_2} \tag{15c}$$

Having determined ψ_1, ψ_2, and ψ_3 the potential of the transmission line is as before

$$P_T = K\psi_1 \log\frac{a'}{a} + K\psi_2 \log\frac{b'}{b} + K\psi_3 \log\frac{c'}{c} \tag{16}$$

CASE III ONE WIRE GROUNDED.

Equation (14) holds in this case as in case II. When one wire is grounded, however, $\psi_1 + \psi_2 + \psi_3$ is no longer zero. The third equation must be obtained from the fact that the potential of A, the grounded wire, is zero. Referring to equation (11)

$$K\psi_1\log\frac{2h_1}{r_1} + K\psi_2\log\frac{A'B}{AB} + K\psi_3\log\frac{A'C}{AC} = 0 \qquad \text{and from (14)}$$

$$K\psi_1\log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} + K\psi_2\log\frac{AC}{A'C}\cdot\frac{A'B}{AB} + K\psi_3\log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} = E_m\sin\omega t$$

$$K\psi_1\log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} + K\psi_2\log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} + K\psi_3\log\frac{AB}{AB'}\cdot\frac{B'C}{BC} = E_m\sin(\omega t+120^\circ) \tag{17}$$

The solution of (10) is:

Determinant of system:

$$\begin{vmatrix} \log\frac{2h_1}{r_1} & \log\frac{A'B}{AB} & \log\frac{A'C}{AC} \\ \log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} & \log\frac{AC}{A'C}\cdot\frac{A'B}{AB} & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} & \log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \end{vmatrix} = D_3 \tag{18}$$

$$K\psi_1 = \frac{E_m\begin{vmatrix} 0 & \log\frac{A'B}{AB} & \log\frac{A'C}{AC} \\ \sin\omega t & \log\frac{AC}{A'C}\cdot\frac{A'B}{AB} & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \sin(\omega t+120^\circ) & \log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \end{vmatrix}}{D_3} \tag{18a}$$

$$K\psi_2 = \frac{E_m\begin{vmatrix} \log\frac{2h_1}{r_1} & 0 & \log\frac{A'C}{AC} \\ \log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} & \sin\omega t & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} & \sin(\omega t+120^\circ) & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \end{vmatrix}}{D_3} \tag{18b}$$

$$K\psi_3 = \frac{\begin{vmatrix} \log\frac{2h_1}{r_1} & \log\frac{A'B}{AB} & 0 \\ \log\frac{A\ B}{A'B}\cdot\frac{2h_1}{r_1} & \log\frac{A\ C}{A'C}\cdot\frac{A'B}{A\ B} & \sin\omega t \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{A\ B} & \log\frac{B\ C}{B'C}\ \frac{2h_2}{r_2} & \sin(\omega t+120^\circ) \end{vmatrix}}{D_3} E_m \qquad (18c)$$

and as before, having determined ψ_1, ψ_2, and ψ_3, the potential of the telephone line is:

$$P_T = K\psi_1 \log\frac{a'}{a} + K\psi_2 \log\frac{b'}{b} + K\psi_3 \log\frac{c'}{c} \qquad (19)$$

CASE IV GROUNDED WIRES.

Let there be added to the conditions of case II ground wires for instance F and G above the line for protection and the telephone lines S and T grounded through drainage coils (assuming practically perfect ground). There exist fluxes now not only upon the line wires but also upon these ground wires -- seven fluxes in all. The seven equations necessary to determine these fluxes are obtained, in the condition where no line wire is grounded, as follows: one from the condition that since the line wires are insulated from ground, the total flux on all three must be zero, two from the voltage relations between line wires (there are three, but only two are independent), and four from the fact that the voltage between each ground wire and its image is zero. The expressions for these voltage relations are given in Table IV. The solution of the equations indicated by the vertical columns of Table IV (together with $\psi_1 + \psi_2 + \psi_3 = 0$) is given in condensed determinant form on page 14. The determinant shown is the determinant of the system written so that coefficients of one unknown are in the same horizontal row instead of the same vertical column. For example, on page 14 the coefficients of $K\psi_1$ are in the first row, whereas on page 10 they

TABLE IV

Flux	At	A & B	B & C	F & F'	G & G'	S & S'	T & T'
		POTENTIAL DIFFERENCE BETWEEN					
ψ_1, $-\psi_1$	A, A'	$K\psi_1 \cdot \log\frac{AB}{r_1}\cdot\frac{2h_1}{A'B}$	$\log\frac{AC}{AB}\cdot\frac{A'C}{AC}$	$\log\frac{AF'}{AF}$	$\log\frac{AG'}{AG}$	$\log\frac{AS'}{AS}$	$\log\frac{AT'}{AT}$
ψ_2, $-\psi_2$	B, B'	$K\psi_2 \cdot \log\frac{r_2}{BA}\cdot\frac{B'A}{2h_2}$	$\log\frac{BC}{r_2}\cdot\frac{2h_2}{B'C}$	$\log\frac{BF'}{BF}$	$\log\frac{BG'}{BG}$	$\log\frac{BS'}{BS}$	$\log\frac{BT'}{BT}$
ψ_3, $-\psi_3$	C, C'	$K\psi_3 \cdot \log\frac{CB}{CA}\cdot\frac{C'A}{C'B}$	$\log\frac{r_3}{CB}\cdot\frac{C'B}{2h_3}$	$\log\frac{CF'}{CF}$	$\log\frac{CG'}{CG}$	$\log\frac{CS'}{CS}$	$\log\frac{CT'}{CT}$
ψ_4, $-\psi_4$	F, F'	$K\psi_4 \cdot \log\frac{FB}{FA}\cdot\frac{F'A}{F'B}$	$\log\frac{FC}{FB}\cdot\frac{F'B}{FC}$	$\log\frac{2h_4}{r_4}$	$\log\frac{FG'}{FG}$	$\log\frac{FS'}{FS}$	$\log\frac{FT'}{FT}$
ψ_5, $-\psi_5$	G, G'	$K\psi_5 \cdot \log\frac{GB}{GA}\cdot\frac{G'A}{G'B}$	$\log\frac{GC}{GB}\cdot\frac{G'B}{GC}$	$\log\frac{GF'}{GF}$	$\log\frac{2h_5}{r_5}$	$\log\frac{GS'}{GS}$	$\log\frac{GT'}{GT}$
ψ_6, $-\psi_6$	S, S'	$K\psi_6 \cdot \log\frac{SB}{SA}\cdot\frac{S'A}{SB}$	$\log\frac{SC}{SB}\cdot\frac{S'B}{S'C}$	$\log\frac{SF'}{SF}$	$\log\frac{SG'}{SG}$	$\log\frac{2h_6}{r_6}$	$\log\frac{ST'}{ST}$
ψ_7, $-\psi_7$	T, T'	$K\psi_7 \cdot \log\frac{TB}{TA}\cdot\frac{T'A}{TB}$	$\log\frac{TC}{TB}\cdot\frac{T'B}{TC}$	$\log\frac{TF'}{TF}$	$\log\frac{TG'}{TG}$	$\log\frac{TS'}{TS}$	$\log\frac{2h_7}{r_7}$
Sum is		$E \sin \omega t$	$E \sin(\omega t+120^{\circ})$	0	0	0	0

NON-GROUNDED

		A - B	B - C	F - F'	G - G'	S - S'	T - T'
$K\psi_1$	1	$\log\frac{B\,A}{B'A}\frac{2h_1}{r_1}$	$\log\frac{C\,A}{C'A}\frac{B'A}{B\,A}$	$\log\frac{F'A}{F\,A}$	$\log\frac{G'A}{G\,A}$	$\log\frac{S'A}{S\,A}$	$\log\frac{T'A}{T\,A}$
$K\psi_2$	1	$\log\frac{r_2}{2h_2}\frac{A'B}{A\,B}$	$\log\frac{C\,B}{C'B}\frac{2h_2}{r_2}$	$\log\frac{F'B}{F\,B}$	$\log\frac{G'B}{G\,B}$	$\log\frac{S'B}{S\,B}$	$\log\frac{T'B}{T\,B}$
$K\psi_3$	1	$\log\frac{B\,C}{B'C}\frac{A'C}{A\,C}$	$\log\frac{r_3}{2h_3}\frac{B'C}{B\,C}$	$\log\frac{F'C}{F\,C}$	$\log\frac{G'C}{G\,C}$	$\log\frac{S'C}{S\,C}$	$\log\frac{T'C}{T\,C}$
$K\psi_4$	0	$\log\frac{B\,F}{B'F}\frac{A'F}{A\,F}$	$\log\frac{C\,F}{C'F}\frac{B'F}{B\,F}$	$\log\frac{2h_4}{r_4}$	$\log\frac{G'F}{G\,F}$	$\log\frac{S'F}{S\,F}$	$\log\frac{T'F}{T\,F}$
$K\psi_5$	0	$\log\frac{B\,G}{B'G}\frac{A'G}{A\,G}$	$\log\frac{C\,G}{C'G}\frac{B'G}{B\,G}$	$\log\frac{F'G}{F\,G}$	$\log\frac{2h_5}{r_5}$	$\log\frac{S'G}{S\,G}$	$\log\frac{T'G}{T\,G}$
$K\psi_6$	0	$\log\frac{B\,S}{B'S}\frac{A'S}{A\,S}$	$\log\frac{C\,S}{C'S}\frac{B'S}{B\,S}$	$\log\frac{F'S}{F\,S}$	$\log\frac{G'S}{G\,S}$	$\log\frac{2h_6}{r_6}$	$\log\frac{T'S}{T\,S}$
$K\psi_7$	0	$\log\frac{B\,T}{B'T}\frac{A'T}{A\,T}$	$\log\frac{C\,T}{C'T}\frac{B'T}{B\,T}$	$\log\frac{F'T}{F\,T}$	$\log\frac{G'T}{G\,T}$	$\log\frac{S'T}{S\,T}$	$\log\frac{2h_7}{r_7}$
	0	$E\sin\omega t$	$E\sin(\omega t+120^\circ)$	0	0	0	0

are in the first column. At the left of this determinant of the system appear opposite each line the unknown of which the terms of that line are coefficients. Above the determinant are indicated the various relations from which each equation (vertical rows are equations) is obtained. Below each column appears the right hand member of the equation of which that row is left hand member. To find the value of any flux, for instance ψ_2, this lower row, (right hand members) must be substituted for the row opposite that flux, ψ_2, and the new determinant thus formed divided by the original determinant (of the system). To illustrate, in solving for ψ_2, the row

$$0 \quad E \sin\omega t \quad E \sin(\omega t+120^\circ) \quad 0 \quad 0 \quad 0 \quad 0$$

would be substituted for

$$1 \quad \log\frac{r_2}{2h_2}\cdot\frac{A'B}{A\ B} \quad \log\frac{C\ B}{C'B}\cdot\frac{2h_2}{r_2} \quad \log\frac{F'B}{FB} \quad \log\frac{G'B}{G\ B} \quad \log\frac{S'B}{S\ B} \quad \log\frac{T'B}{T\ B}$$

in the dividend determinant.

An exactly similar set of determinants for the case where wire A is grounded appears on page 17. It differs from that on page 13 by the first columns only ($A - A' = 0$ instead of $\psi_1 + \psi_2 + \psi_3 = 0$).

The determinants for the case of grounded neutral are on page 18. This differs from the other two in the first three columns ($A - A' = 2e\sin(\omega t+120^\circ)$, $B - B' = 2e \sin(\omega t+120^\circ)$, $C - C' = 2e \sin(\omega t+240^\circ)$ instead of $\psi_1 + \psi_2 + \psi_3 = 0$, $A - B = E \sin \omega t$, $B - C = E \sin(\omega t + 120^\circ)$). No further description of these is necessary, and their use has been sufficiently explained above.

The determinants on pages 14, 17, and 18 may easily be changed to fit any case where only one three phase power line is involved. If, for instance, only one overhead ground wire exists, G, we may strike out the row of ψ_4 and the column of F-F', and the remainder of the determinant represents the new conditions. For we may

consider F and G as the same wire. Our equations will, then, be of the form

$$\left.\begin{array}{l} 4u + x + y + z = a \\ 2u + x + y + 2z = b \\ 2u + x + y + 2z = b \\ u + x + y + 3z = c \end{array}\right)$$

Obviously there are only three equations and only three unknowns, u, x + y, and z. If now, we write x for x + y the equations become

$$\left.\begin{array}{l} 4u + x + z = a \\ 2u + x + 2z = b \\ u + x + 3z = c \end{array}\right)$$

which are three equations in three unknowns, and hence soluble. But the same result would have been obtained by striking out the y column and the third equation.

To extend the determinant to a greater number of ground wires requires the addition of as many rows and columns as there are additional wires. These are, however, easily written from symmetry. If the added wire be, for example, J, of radius r_8, height h_8, carrying flux ψ_8, and the non-grounded case case be desired, the added lines will be those on page 19. The determinant may be extended in this way as far as necessary so long as only one three phase power circuit exists.

The voltage of any desired point is found as before by adding the effects of all the fluxes:

$$P_T = K_1 \log\frac{a'}{a} + K_2 \log\frac{b'}{b} + K_3 \log\frac{c'}{c} + K_4 \log\frac{f'}{f}$$
$$+ K_5 \log\frac{g'}{g} + K_6 \log\frac{s'}{s} + K_7 \log\frac{t'}{t} + \text{etc.}$$

where a' is distance of point from A'

a " " " " " A

WIRE "A" GROUNDED

	A-A'	A-B	B-C	F-F'	G-G'	S-S'	T-T'
$K\psi_1$	$\log\frac{2h_1}{r_1}$	$\log\frac{BA}{B'A}\frac{2h_1}{r_1}$	$\log\frac{CA}{C'A}\frac{B'A}{BA}$	$\log\frac{F'A}{FA}$	$\log\frac{G'A}{GA}$	$\log\frac{S'A}{SA}$	$\log\frac{T'A}{TA}$
$K\psi_2$	$\log\frac{A'B}{AB}$	$\log\frac{r_2}{2h_2}\frac{A'B}{AB}$	$\log\frac{CB}{C'B}\frac{2h_2}{r_2}$	$\log\frac{F'B}{FB}$	$\log\frac{G'B}{GB}$	$\log\frac{S'B}{SB}$	$\log\frac{T'B}{TB}$
$K\psi_3$	$\log\frac{A'C}{AC}$	$\log\frac{BC}{B'C}\frac{A'C}{AC}$	$\log\frac{r_3}{2h_3}\frac{B'C}{BC}$	$\log\frac{F'C}{FC}$	$\log\frac{G'C}{GC}$	$\log\frac{S'C}{SC}$	$\log\frac{T'C}{TC}$
$K\psi_4$	$\log\frac{A'F}{AF}$	$\log\frac{BF}{B'F}\frac{A'F}{AF}$	$\log\frac{CF}{C'F}\frac{B'F}{BF}$	$\log\frac{2h_4}{r_4}$	$\log\frac{G'F}{GF}$	$\log\frac{S'F}{SF}$	$\log\frac{T'F}{TF}$
$K\psi_5$	$\log\frac{A'G}{AG}$	$\log\frac{BG}{B'G}\frac{A'G}{AG}$	$\log\frac{CG}{C'G}\frac{B'G}{BG}$	$\log\frac{F'G}{FG}$	$\log\frac{2h_5}{r_5}$	$\log\frac{S'G}{SG}$	$\log\frac{T'G}{TG}$
$K\psi_6$	$\log\frac{A'S}{AS}$	$\log\frac{BS}{B'S}\frac{A'S}{AS}$	$\log\frac{CS}{C'S}\frac{B'S}{BS}$	$\log\frac{F'S}{FS}$	$\log\frac{G'S}{GS}$	$\log\frac{2h_6}{r_6}$	$\log\frac{T'S}{TS}$
$K\psi_7$	$\log\frac{A'T}{AT}$	$\log\frac{BT}{B'T}\frac{A'T}{AT}$	$\log\frac{CT}{C'T}\frac{B'T}{BT}$	$\log\frac{F'T}{FT}$	$\log\frac{G'T}{GT}$	$\log\frac{S'T}{ST}$	$\log\frac{2h_7}{r_7}$
	0	$E\sin\omega t$	$E\sin(\omega t+120^\circ)$	0	0	0	0

NEUTRAL GROUNDED

	A-A'	B-B'	C-C'	F-F'	G-G'	S-S'	T-T'
$K\psi_1$	$\log\frac{2h_1}{r_1}$	$\log\frac{B'A}{B\ A}$	$\log\frac{C'A}{C\ A}$	$\log\frac{F'A}{F\ A}$	$\log\frac{G'A}{G\ A}$	$\log\frac{S'A}{S\ A}$	$\log\frac{T'A}{T\ A}$
$K\psi_2$	$\log\frac{A'B}{A\ B}$	$\log\frac{2h_2}{r_2}$	$\log\frac{C'B}{C\ B}$	$\log\frac{F'B}{F\ B}$	$\log\frac{G'B}{G\ B}$	$\log\frac{S'B}{S\ B}$	$\log\frac{T'B}{T\ B}$
$K\psi_3$	$\log\frac{A'C}{A\ C}$	$\log\frac{B'C}{B\ C}$	$\log\frac{2h_3}{r_3}$	$\log\frac{F'C}{F\ C}$	$\log\frac{G'C}{G\ C}$	$\log\frac{S'C}{S\ C}$	$\log\frac{T'C}{T\ C}$
$K\psi_4$	$\log\frac{A'F}{A\ F}$	$\log\frac{B'F}{B\ F}$	$\log\frac{C'F}{C\ F}$	$\log\frac{2h_4}{r_4}$	$\log\frac{G'F}{G\ F}$	$\log\frac{S'F}{S\ F}$	$\log\frac{T'F}{T\ F}$
$K\psi_5$	$\log\frac{A'G}{A\ G}$	$\log\frac{B'G}{B\ G}$	$\log\frac{C'G}{C\ G}$	$\log\frac{F'G}{F\ G}$	$\log\frac{2h_5}{r_5}$	$\log\frac{S'G}{S\ G}$	$\log\frac{T'G}{T\ G}$
$K\psi_6$	$\log\frac{A'S}{A\ S}$	$\log\frac{B'S}{B\ S}$	$\log\frac{C'S}{C\ S}$	$\log\frac{F'S}{F\ S}$	$\log\frac{G'S}{G\ S}$	$\log\frac{2h_6}{r_6}$	$\log\frac{T'S}{T\ S}$
$K\psi_7$	$\log\frac{A'T}{A\ T}$	$\log\frac{B'T}{B\ T}$	$\log\frac{C'T}{C\ T}$	$\log\frac{F'T}{F\ T}$	$\log\frac{G'T}{G\ T}$	$\log\frac{S'T}{S\ T}$	$\log\frac{2h_7}{r_7}$
	$e\sin\omega t$	$e\sin(\omega t+120^\circ)$	$e\sin(\omega t+240^\circ)$	0	0	0	0

		A - B	B - C	F - F'	G - G'	S - S'	T - T'	J - J'
$K\psi_1$								$\log\frac{J'A}{J\ A}$
$K\psi_2$								$\log\frac{J'B}{J\ B}$
$K\psi_3$								$\log\frac{J'C}{J\ C}$
$K\psi_4$								$\log\frac{J'F}{J\ F}$
$K\psi_5$								$\log\frac{J'G}{J\ G}$
$K\psi_6$								$\log\frac{J'S}{J\ S}$
$K\psi_7$								$\log\frac{J'T}{J\ T}$
$K\psi_8$	0	$\log\frac{B\ J}{B'J}\frac{A'J}{A\ J}$	$\log\frac{C\ J}{C'J}\frac{B'J}{B\ J}$	$\log\frac{F'J}{F\ J}$	$\log\frac{G'J}{G\ J}$	$\log\frac{S'J}{S\ J}$	$\log\frac{T'J}{T\ J}$	$\log\frac{2hc}{r_8}$
	0	$E\sin\omega t$	$E\sin(\omega t+120^\circ)$	0	0	0	0	0

b'	is	distance	of	point	from	B'
b	"	"	"	"	"	B
c'	"	"	"	"	"	C'
c	"	"	"	"	"	C
f'	"	"	"	"	"	F'
f	"	"	"	"	"	F
g'	"	"	"	"	"	G'
s'	"	"	"	"	"	S'
s	"	"	"	"	"	S
t'	"	"	"	"	"	T'
t	"	"	"	"	"	T

CHARGING CURRENT

No more than the foregoing is needed for the determination of the potential distribution about any ordinary single power circuit line. A trifling addition will make the determinants applicable for the calculation of charging current as well. This charging current is $I_c = \frac{d\psi}{dt}$. Now ψ is a quantity containing $\sin\omega t$, $\sin(\omega t+120^\circ)$, $\sin(\omega t+240^\circ)$. This may be transformed into the form $\Psi\sin(\omega t+x)$ whence

$$I_c = \frac{d\psi}{dt} = \omega\Psi\cos(\omega t+x)$$

It remains to find Ψ from $K\Psi$ which is given by the equations. Changing the equation (8) the capacity of the thin ring 1 cm long of radius x cm is

$$\frac{\psi}{de} = \frac{\text{area}}{\text{length}} \cdot \text{absolute permittivity}$$

$$= \frac{2\pi x}{dx} \cdot .08842 \cdot 10^{-6} \text{*} \tag{21}$$

Changing again to the form of (8)

$$\frac{de}{dx} = \frac{\psi}{2\pi x} \cdot \frac{10^6}{.08842} \tag{22}$$

* Karapetoff: "The Electric Current."

which by comparison with (8) gives

$$K_1 = \frac{10^6}{.08842} \tag{23}$$

From the substitution in (9)

$$K = \frac{2.3K_1}{2\pi} = \frac{2.3\cdot 10^6}{6.28\cdot .08842} = 4.14\cdot 10^6 \tag{24}$$

Hence the value of ψ obtained by means of the determinants should be divided by $4.14 \cdot 10^6$. The value of Ψ so obtained will give amperes in (20).

It should be noted that, whereas to find the potential of any point it is necessary to find all the flux values, the charging current in any line is found by solving for the flux on that line alone.

In cases where one of the power lines is grounded, the sum of the fluxes on the three line wires is not zero. This resultant flux exists on the ground, and from it may be found the charging current in the ground.

CALCULATIONS:

In general the determinants may be somewhat simplified before calculation. As example take equation 15, (also 15a,b,c.)

$$D_2 = \begin{vmatrix} \log\frac{AB}{A'B}\cdot\frac{2h_1}{r_1} & \log\frac{AC}{A'C}\cdot\frac{A'B}{AB} & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{AC} \\ \log\frac{r_2}{2h_2}\cdot\frac{A'B}{AB} & \log\frac{BC}{B'C}\cdot\frac{2h_2}{r_2} & \log\frac{AB}{A'B}\cdot\frac{B'C}{BC} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log\frac{AB}{A'B}\,\frac{2h_1}{r_1} & \log\frac{AC}{A'C}\left\{\frac{A'B}{AB}\right\}^2\cdot\frac{r_1}{2h_1} & \log\left\{\frac{r_1}{2h_1}\right\}^2\cdot\frac{A'C}{AC}\cdot\frac{A'B}{AB} \\ \log\frac{r_2}{2h_2}\,\frac{A'B}{AB} & \log\frac{BC}{B'C}\left\{\frac{2h_2}{r_2}\right\}^2\cdot\frac{A'B}{AB} & \log\left\{\frac{AB}{A'B}\right\}^2\frac{B'C}{BC}\,\frac{2h_2}{r_2} \\ 1 & 0 & 0 \end{vmatrix}$$

(Subtracting column 1 from columns 2 and 3).

$$D_2 = 1 \begin{vmatrix} \log\frac{A\ C}{A'C}\cdot\left\{\frac{A'B}{A\ B}\right\}^2\cdot\frac{r_1}{2h_1} & \log\left\{\frac{r_1}{2h_1}\right\}^2\cdot\frac{A'C}{A\ C}\cdot\frac{A'B}{A\ B} \\ \log\frac{B\ C}{B'C}\cdot\left\{\frac{2h_2}{r_2}\right\}^2\cdot\frac{A'B}{A\ B} & \log\left\{\frac{A\ B}{A'B}\right\}^2\cdot\frac{B'C}{B\ C}\cdot\frac{2h_2}{r_2} \end{vmatrix}$$

(taking minors on bottom row)

From (15a)

$$K\psi_1 = \frac{E_m}{D_2}\begin{vmatrix} \sin\omega t & \log\frac{A\ C}{A'C}\cdot\frac{A'B}{A\ B} & \log\frac{r_1}{2h_1}\cdot\frac{A'C}{A\ C} \\ \sin(\omega t+120^\circ) & \log\frac{B\ C}{B'C}\cdot\frac{2h_2}{r_2} & \log\frac{A\ B}{A'B}\cdot\frac{B'C}{B\ C} \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \frac{E_m}{D_2}\left\{ \log\left\{\frac{B\ C}{B'C}\right\}^2\cdot\frac{2h_2}{r_2}\cdot\frac{A'B}{A\ B}\ \sin\omega t + \log\frac{r_1}{2h_1}\cdot\left\{\frac{A'C}{A\ C}\right\}^2\cdot\frac{A\ B}{A'B}\ \sin(\omega t+120^\circ)\right\}$$

Similar forms will be obtained for $K\psi_2$ and $K\psi_3$. It is seen that each flux is given as a function of $\sin\omega t$ and $\sin(\omega t+120^\circ)$, and also, in the neutral grounded case, of $\sin(\omega t+240^\circ)$. These may always be added to give a term like

$$K\psi = K\Psi\ \sin(\omega t+x)$$

Ψ and x may be best determined by a graphical vector addition of the coefficients of $\sin\omega t$, etc. Thus suppose

$$K\psi = L\sin\omega t + M\sin(\omega t+120^\circ) + N\sin(\omega t+240^\circ)$$

$$= K\Psi\sin(\omega t+x)$$

Then Fig. 5 shows how $K\Psi$ and x may be obtained.

Where N is 0 as is the case when the neutral is not grounded this direct diagram is best. When, however, the neutral is grounded and all the sine terms exist, it is simpler to use L-M and M-N as indicated in Fig 5 and shown to a larger scale in Fig 6.

Similar additions may be performed in the case of finding the voltage at a point. For this, however, it is well to tabulate the calculations. A sample table is given for the case of grounded neutral.

Suppose that

$K\psi_1$ is $= L_1 \sin\omega t + M_1 \sin(\omega t+120^\circ) + N_1 \sin(\omega t+240^\circ)$

$K\psi_2$ is $= L_2 \sin\omega t + M_2 \sin(\omega t+120^\circ) + N_2 \sin(\omega t+240^\circ)$

$K\psi_3$ is $= L_3 \sin\omega t + M_3 \sin(\omega t+120^\circ) + N_2 \sin(\omega t+240^\circ)$

etc etc

We may tabulate

Col.1	Col.2	Col.3
$L_1 \log\frac{a'}{a}$	$M_1 \log\frac{a'}{a}$	$N_1 \log\frac{a'}{a}$
$L_2 \log\frac{b'}{b}$	$M_2 \log\frac{b'}{b}$	$N_2 \log\frac{b'}{b}$
$L_3 \log\frac{c'}{c}$	$M_3 \log\frac{c'}{c}$	$N_3 \log\frac{c'}{c}$
etc	etc	etc

The sums of these columns are

L	M	N

which are to be used in Fig 5.or Fig.6.

The ψ or P_T obtained as here indicated will be the maximum value of a sine wave. The effective value which is generally desired may be obtained by dividing by 1.414; or, if instead of maximum values E_m and e_m the effective values E and e be used, the result will be effective value automatically.

HKH:GCH

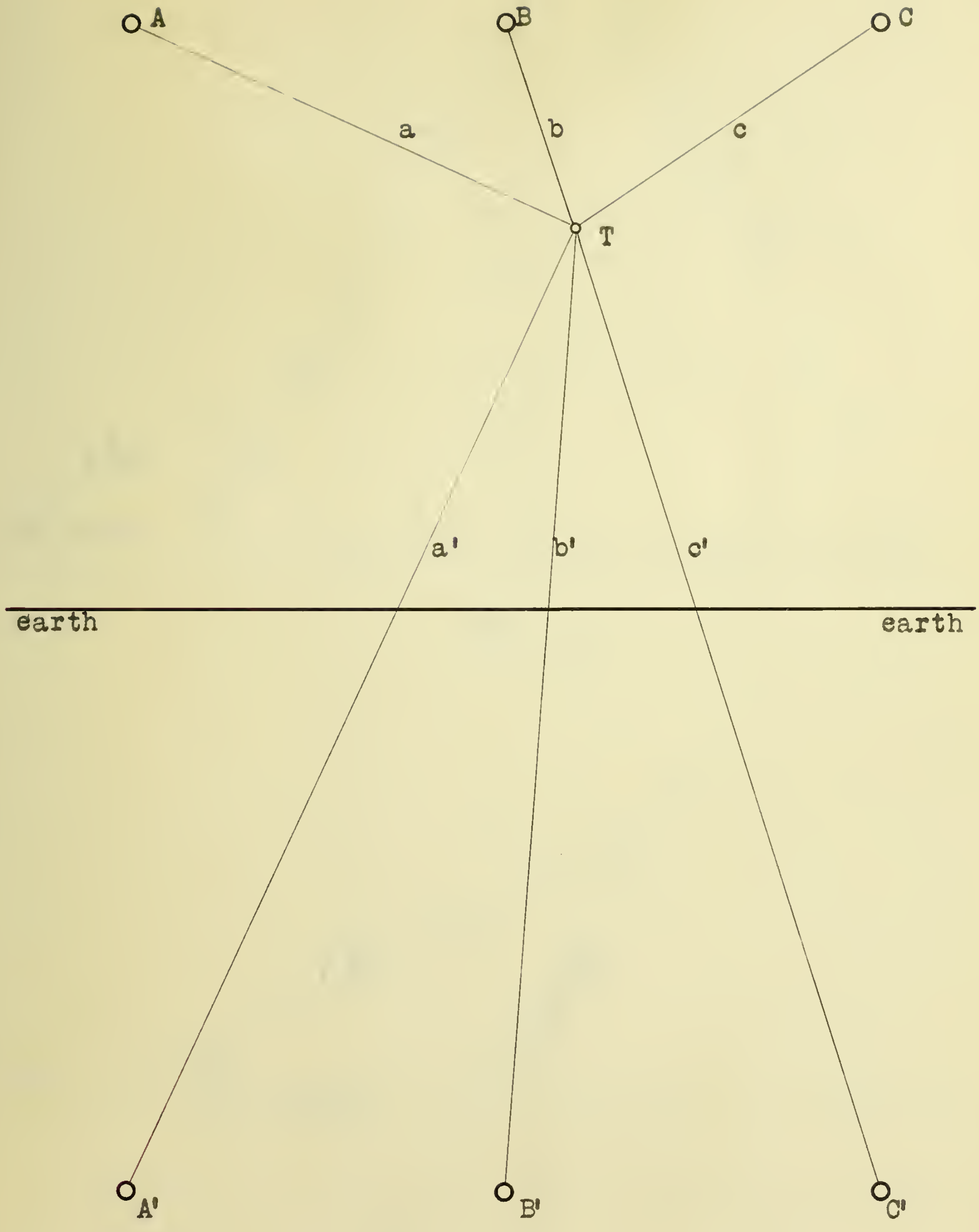

Fig. 4

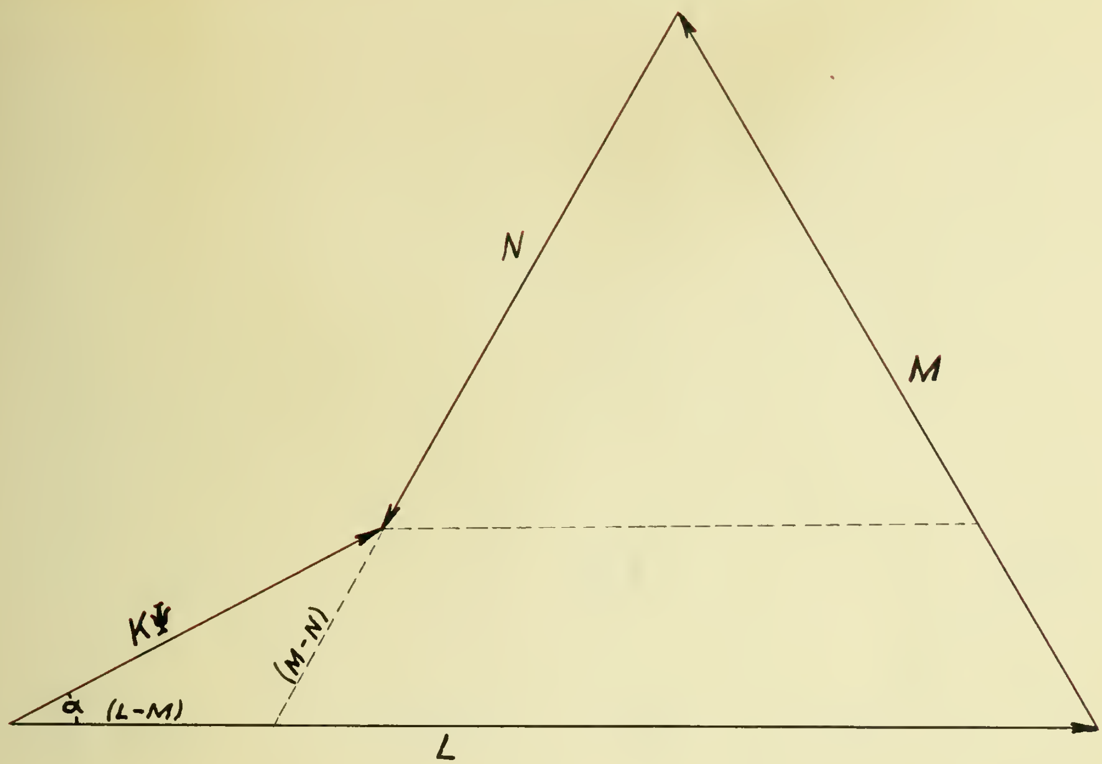

Fig. 5

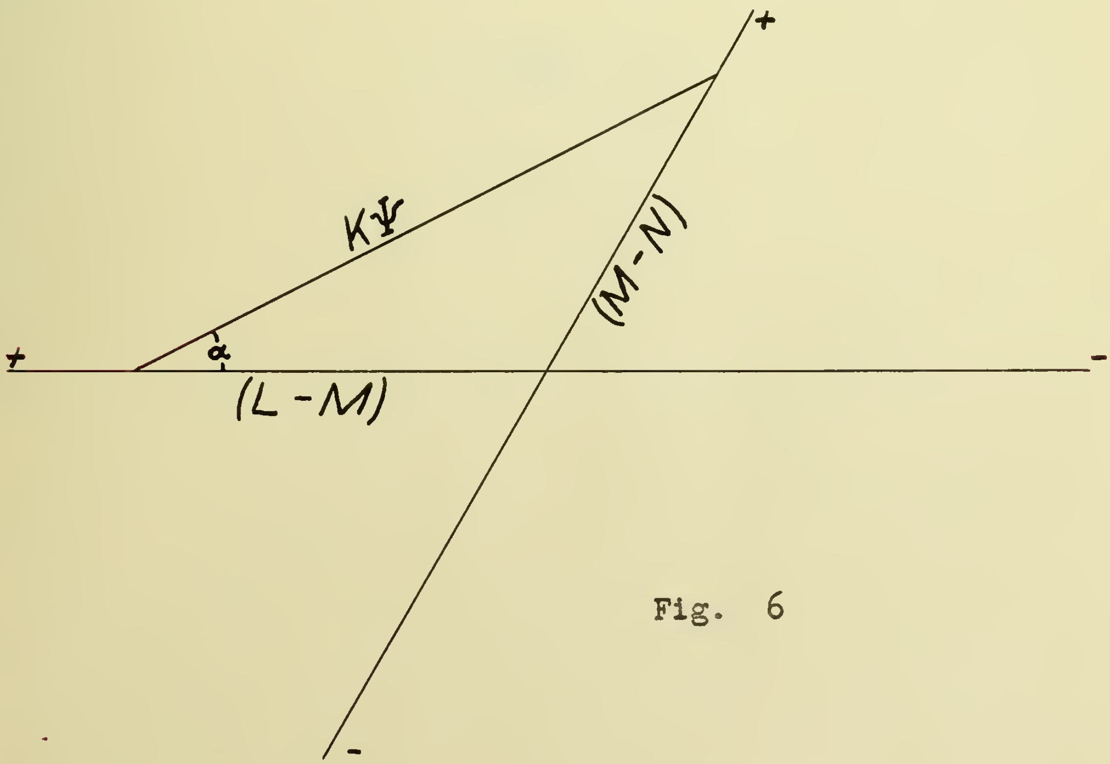

Fig. 6

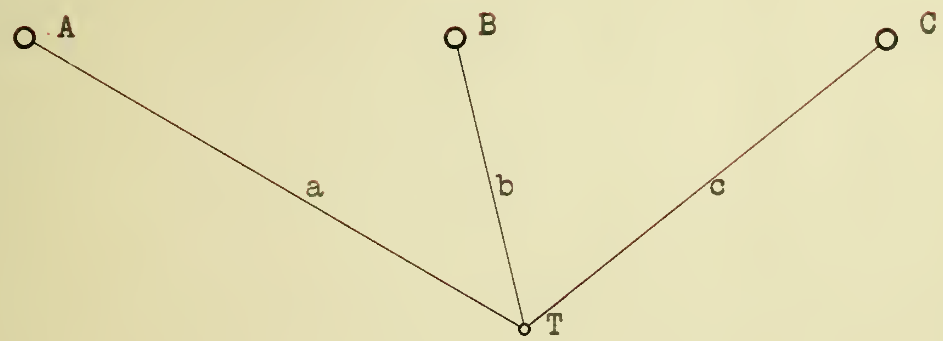

Fig. 1